UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

LEVEL IV

(2)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: 030
2. GOVT ACCESSION NO.: AD-A109 329
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): Longitude Determination by Two Different Stars Observed at the Same Altitude East and West of the Meridian
5. TYPE OF REPORT & PERIOD COVERED: Paper
6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): Dr. Angel A. Baldini
8. CONTRACT OR GRANT NUMBER(s):
9. PERFORMING ORGANIZATION NAME AND ADDRESS:
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS:
11. CONTROLLING OFFICE NAME AND ADDRESS: US Army Engineer Topographic Laboratories, Ft. Belvoir, VA 22060
12. REPORT DATE: 14 Mar 81
13. NUMBER OF PAGES: 13
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS. (of this report):
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report): Approved for Public Release; distribution unlimited
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
18. SUPPLEMENTARY NOTES

DTIC ELECTE JAN 5 1982 S D

D

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
determination of time
observation of
zenith distance
prime vertical

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The determination of time by east and west observation of the zenith distance of a star near the prime vertical shows discrepancies as much as 2 seconds of time. A new method with higher accuracy has been developed. It is based on observation of an individual pair of stars. The observation of each star (one east, the other west of the meridian) consists of determining the instant of time at which it crosses a fixed preselected unknown zenith in order to determine the circumstances when greatest accuracy can be expected.

AD A109329

DTIC FILE COPY

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED 403192
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

LONGITUDE DETERMINATION BY TWO DIFFERENT STARS OBSERVED AT THE SAME ALTITUDE EAST AND WEST OF THE MERIDIAN

Dr. Angel A. Baldini
U.S. Army Engineer Topographic Laboratories
Fort Belvoir, VA 22060

BIOGRAPHICAL SKETCH

Dr. Angel A. Baldini joined ETL's predecessor organization in 1960. From 1957-1960 he was associated with the Georgetown University Observatory, Washington, D.C. Prior to 1957, he was Professor and Head, Department of Geodesy, La Plata University, Argentina. At ETL, Dr. Baldini works primarily in the field of astro-geodesy. Since 1974 he has been senior scientist in the Center for Geodesy, Research Institute, ETL. He authored 36 reports and papers since 1963 and presented results thereof at 20 Army, national and international meetings. He received an Army Research and Development achievement award in 1969. He has been a member of the American Geophysical Union since 1960.

ABSTRACT

The determination of time by east and west observation of the zenith distance of a star near the prime vertical shows discrepancies as much as 2 seconds of time.

A new method with higher accuracy has been developed. It is based on observation of an individual pair of stars. The observation of each star (one east, the other west of the meridian) consists of determining the instant of time at which it crosses a fixed preselected unknown zenith distance. Complete analysis of this method was undertaken in order to determine the circumstances when greatest accuracy can be expected.

In this method the longitude or clock correction is obtained as a function of the recorded instant of time when two known stars, one to the west the other to the east of the meridian, cross a fixed preselected zenith distance. The instrumental adjustment must not change between the two observations. Changes may occur due to the conditions of the atmosphere and temperature variations, therefore changes in refraction may also occur during the period between the two observations. But corrections for this matter can be applied. In general the elapsed time between the two observations is short, no more than a few minutes, so no correction for refraction is needed.

To obtain higher accuracy the two stars are to be selected with not too large a difference in their declinations, and they must be observed close to the prime vertical because the velocity of variation in the zenith distance is a maximum in the prime vertical consequently making it possible to obtain a higher accuracy in recording the time of passages over the horizontal line.

Let figure 1 represent the projection of the celestial sphere on the horizontal plane. In this figure, Z is the projection of the zenith of the observing station, and P_n, the North Pole projection. N-S is the meridian and W-E the prime vertical. The small circle with center at Z is the almucantarat, parallel to the horizon, SEN.

Let S_1, be the star to the east of the meridian and S_2, the star to the west of it. The zenith distance is

$$z = ZS_1 = ZS_2$$

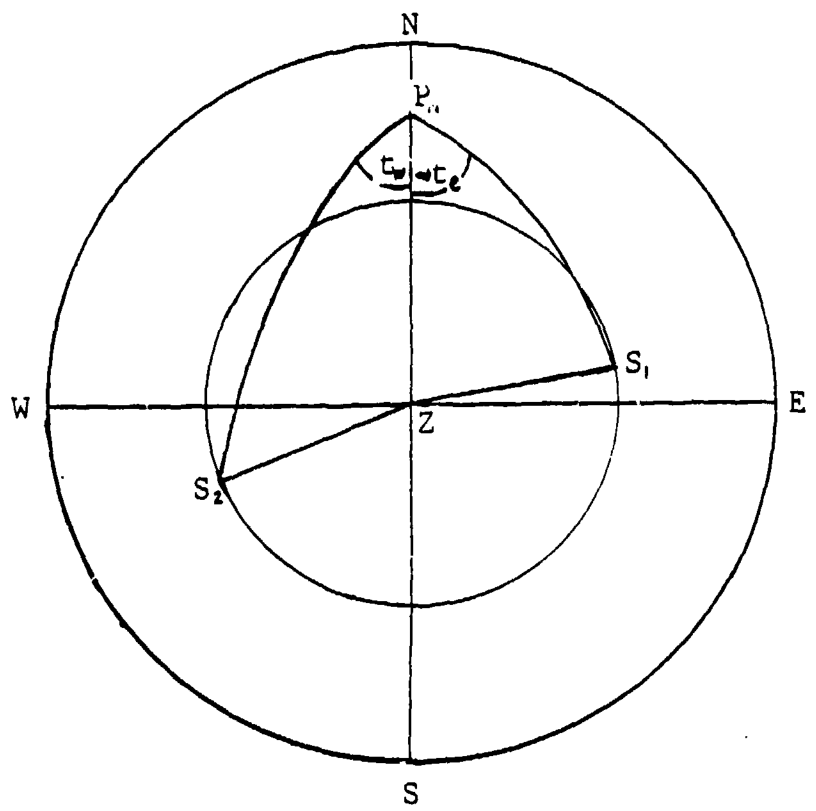

Figure 1, projection on the horizontal plane.

Let

θ_w, θ_e = the observed sideral times

α_e, δ_e = the right ascension and declination of the east star, S_1.

α_w, δ_w = the right ascension and declination of the west star, S_2.

t_e, t_w = the hour angles of S_1 and S_2 , respectively.

ϕ = latitude

In the astronomical triangles $ZP_n S_1$ and $ZP_n S_2$, from formula of cosine, we have

$$\begin{aligned} \cos Z &= \sin\phi \sin\delta_e + \cos\phi \cos\delta_e \cos t_e \\ \cos Z &= \sin\phi \sin\delta_w + \cos\phi \cos\delta_w \cos t_w . \end{aligned} \tag{1}$$

Introducing the parameters

$$\begin{aligned} X_1 &= \sin\phi \sin\delta_e \\ X_2 &= \sin\phi \sin\delta_w \end{aligned} \tag{2}$$

$$\begin{aligned} Y_1 &= \cos\phi \cos\delta_e \\ Y_2 &= \cos\phi \cos\delta_w , \end{aligned} \tag{3}$$

equations (1) can be rewritten in a more compact notation as follows:

$$\begin{aligned} \cos Z &= X_1 + Y_1 \cos t_e \\ \cos Z &= X_2 + Y_2 \cos t_w . \end{aligned} \tag{4}$$

We can consider either the clock correction or the longitude as unknown and will first consider the case where the clock correction is unknown.

DETERMINATION OF CLOCK CORRECTION

Let ΔT be the clock correction and μ the rate of the clock in units of clock time.

If we put

$$\begin{aligned} \beta_e &= \theta_e + \mu(\theta_e - \theta_0) - \alpha_e \\ \beta_w &= \theta_w + \mu(\theta_w - \theta_0) - \alpha_w , \end{aligned} \tag{5}$$

we shall have

$$\begin{aligned} t_e &= \beta_e + \Delta T \\ t_w &= \beta_w + \Delta T \end{aligned} \tag{6}$$

for the star's hour angle.

Let a parameter τ be defined such that it satisfies the equation

$$\beta_e + \beta_w + 2\tau = 0, \tag{7}$$

thus

$$\tau = -\tfrac{1}{2}(\beta_e + \beta_w). \tag{8}$$

Adding and subtracting τ to each of the equations (6) we obtain

$$\begin{aligned} t_e &= \beta_e + \tau + (\Delta T - \tau) \\ t_w &= \beta_w + \tau + (\Delta T - \tau). \end{aligned} \tag{9}$$

But

$$\begin{aligned} \beta_e + \tau &= \tfrac{1}{2}(\beta_e - \beta_w) \\ \beta_w + \tau &= \tfrac{1}{2}(\beta_w - \beta_e) \end{aligned} \tag{10}$$

and

Setting

$$\beta_o = \tfrac{1}{2}(\beta_w - \beta_e) \tag{11}$$

and

$$\gamma = \Delta T - \tau \tag{12}$$

equations (9) become

$$\begin{aligned} t_e &= -\beta_o + \gamma \\ t_w &= \beta_o + \gamma. \end{aligned} \tag{13}$$

Introducing these values of t_e and t_w into equations (4), developing its cosines and subtracting the first equation from the second, we obtain

$$X_2 - X_1 + (Y_2 - Y_1)\cos\gamma\cos\beta_o - (Y_2 + Y_1)\sin\beta\sin\gamma = 0. \tag{14}$$

To solve this equation let us introduce the auxiliary quantities defined by

$$\begin{aligned} f\sin F &= (Y_2 - Y_1)\cos\beta_o \\ f\cos F &= (Y_2 + Y_1)\sin\beta_o. \end{aligned} \tag{15}$$

Dividing the first by the second we get

$$\tan F = \frac{Y_2 - Y_1}{Y_2 + Y_1}\cot\beta_o. \tag{16}$$

From equation (3) it follows that

$$\frac{Y_2 - Y_1}{Y_2 + Y_1} = \frac{\cos\delta_w - \cos\delta_e}{\cos\delta_w + \cos\delta_e}, \tag{17}$$

whereby equation (16) becomes

$$\tan F = \cot\beta_o(\cos\delta_w - \cos\delta_e)/(\cos\delta_w + \cos\delta_e). \tag{18}$$

Inserting the values shown in equations (15) into equation (14), we find

$$X_2 - X_1 + f \sin(F-\gamma) = 0. \tag{19}$$

From equation (2) we have

$$X_2 - X_1 = \sin\phi(\sin\delta_w - \sin\delta_e) \tag{20}$$

from the second equation (15),

$$f = (Y_2 + Y_1)\sin\beta_0/\cos F \tag{21}$$

and from equation (3),

$$Y_2 + Y_1 = \cos\phi(\cos\delta_w + \cos\delta_e). \tag{22}$$

With these equations, (20, (21), and (22), we finally obtain

$$\sin(\gamma - F) = \tan\tfrac{1}{2}(\delta_w - \delta_e)\tan\phi\cos F\,\mathrm{cosec}\,\beta_0 \tag{23}$$

from equation (19).

Because the stars are selected not far from the prime vertical, $(\gamma-F)$ will always be small and therefore it is accurately defined by equation (23).

Let

$$\gamma - F = \psi, \tag{24}$$

so that

$$\gamma = \psi + F, \tag{25}$$

and from equation (12)

$$\gamma = \Delta T - \tau \tag{12}$$

we obtain the clock correction

$$\Delta T = \psi + F + \tau. \tag{26}$$

Summarizing, the final formulas for computing the clock correction are:

$$\beta_e = \Theta_e + \mu(\Theta_e - \Theta_0) - \alpha_e,$$
$$\beta_w = \Theta_w + \mu(\Theta_w - \Theta_0) - \alpha_w, \tag{5}$$

$$\tau = -\tfrac{1}{2}(\beta_e + \beta_w) \tag{8}$$

$$\beta_0 = \tfrac{1}{2}(\beta_w - \beta_e), \tag{11}$$

$$\tan F = \frac{\cos\delta_w - \cos\delta_e}{\cos\delta_w + \cos\delta_e}\cot\beta_0, \tag{18}$$

$$\sin\psi = \tan\tfrac{1}{2}(\delta_w - \delta_e)\tan\phi\cos F\,\mathrm{cosec}\,\beta_0, \tag{23}$$

$$\psi = \gamma - F, \tag{24}$$

$$\Delta T = F + \gamma + \tau. \tag{25}$$

When tan F or sin $(\gamma-F)$, is positive, F and $(\gamma-F)$ fall in the first quadrant, and when negative, they fall in the fourth quadrant.

The motion of the observer due to the diurnal rotation of the Earth causes all stars apparently to increase in right ascension and decrease in declination. The amount of correction is defined by the well known equations:

$$\Delta\alpha = 0^{s}.021 \cos\phi \cos t \sec\delta \tag{26}$$
$$\Delta\delta = -0''.32 \cos\phi \sin t \sin\delta \tag{27}$$

therefore, the star's hour angles must be corrected for these variations. To obtain these corrections we proceed as follows:

From formula (1) we obtain:

$$\cos\phi \cos t = (\cos z - \sin\phi \sin\delta)\sec\delta \tag{28}$$
$$\cos\phi \cos\delta \sin t = \pm\sqrt{\cos^2\phi \cos^2\delta - (\cos z - \sin\phi \sin\delta)^2} \tag{29}$$

from which equations (26) become

$$\Delta\alpha = 0^{s}.021 (\cos z - \sin\phi \sin\delta)(1+\tan^2\delta) \tag{30}$$

$$\Delta\delta = \pm 0''.32 \tan\delta \sqrt{\cos^2\phi \cos^2\delta - (\cos z - \sin\phi \sin\delta)^2} \tag{31}$$

where the sign must be taken to be positive for the eastern transit and negative for western transit.

Therefore the star coordinates to be considered are:

$$\begin{aligned} &\alpha + \Delta\alpha \\ &\delta + \Delta\delta \end{aligned} \tag{32}$$

So far we have considered how the clock correction can be determined. Let us now consider the case where the longitude is to be determined.

In this case the observation time, based on Greenwich Sidereal Time, will be represented by θ_g and λ will be the longitude, considered positive to the west. In this event the star's transits times will be given by:

θ_{g_e} = east star transit time

θ_{g_w} = west star transit time.

Let λ_0 be an approximate value of the longitude and $d\lambda$ the amount to b added to obtain

$$\lambda = \lambda_0 + \Delta\lambda \, . \tag{33}$$

These equations (5) now become

$$\beta_e = \theta_{g_e} - (\lambda_0 + \alpha_e) + \mu(\theta_{g_e} - \theta_0)$$
$$\beta_w = \theta_{g_w} - (\lambda_0 + \alpha_w) + \mu(\theta_{g_w} - \theta_0) \, . \tag{34}$$

The angles τ and β_0 remain as before

$$\tau = -\frac{1}{2}(\beta_e + \beta_w) \tag{35}$$

$$\beta_0 = \frac{1}{2}(\beta_w - \beta_e) \, . \tag{36}$$

Therefore:

$$\beta_e = -(\beta_0 + \tau)$$
$$\beta_w = \beta_0 - \tau \,, \tag{37}$$

and the hour angles become

$$t_e = -\beta_0 - (\tau + \Delta\lambda) \tag{38}$$
$$t_w = \beta_0 - (\tau + \Delta\lambda) \,.$$

Now by setting

$$\gamma = -(\tau + \Delta\lambda)$$

we get expressions of the same format as those shown in equation (13),

$$t_e = -\beta_0 + \gamma$$
$$t_w = \beta_0 + \gamma \,. \tag{39}$$

From here on the angle F is computed from equation (18); the angle ($\gamma - F$) by equation (23), and the longitude correction $\Delta\lambda$ from

$$\Delta\lambda = -(F + \psi + \tau) \,. \tag{40}$$

Let us now investigate under what conditions an accurate result is to be expected. By differentiating equation (25) assuming that the rate of the clock is sufficiently well known, we have

$$d(\Delta T) = dF + d\psi + d\tau \,. \tag{41}$$

By differentiating equation (23) relative to ϕ, F and β_0, we find

$$d\psi = \frac{2\tan\psi}{\sin 2\phi}\, d\phi - \tan\psi \tan F\, dF - \tan\psi \cot\beta_0\, d\beta_0 . \tag{42}$$

We have, from equation (18)

$$dF = \frac{\sin 2F}{\sin 2\beta_0}\, d\beta_0 \,, \tag{43}$$

from equations (11) and (5), we have

$$d\beta_0 = \frac{1}{2}(d\theta_w - d\theta_e) \,, \tag{44}$$

and from equations (8) and (5),

$$d\tau = -\frac{1}{2}(d\theta_w + d\theta_e) \,.$$

Inserting the values shown in equations (42), (43) and (45) into equation (41), we find

$$d(\Delta T) = \frac{2\tan\psi}{\sin 2\phi}\, d\phi + \left[\tan\psi\left(\frac{\sin 2F}{\sin 2\beta_0} - \cot\beta_0\right) - 1\right]\frac{d\theta_w}{2}$$
$$- \left[\tan\psi\left(\frac{\sin 2F}{\sin 2\phi} - \cot\beta_0\right) + 1\right]\frac{d\theta_e}{2} \,. \tag{46}$$

The angle ψ is always small and reduces to zero at the equator, as can be seen from equation (23), and the angle F also is small (see equation 18). Since the coefficients in parentheses are almost equal to unity, the main source of errors is due to inaccuracy in recording the transit times.

We may observe the star's passage over different horizontal

lines and, so that, we have as many independent results as pairs of lines. The mean times of observation will not be the mean of times at which the star passes the mean zenith distance, so a correction to the mean time is required. We treat this problem independently of the knowledge of the space between lines. The computation is, thus, free of refraction error.

USE OF A ZENITH DISTANCE THAT CORRESPONDS TO THE MEAN STAR TRANSIT TIMES BETWEEN A PAIR OF HORIZONTAL LINES

A theodolite may be used as an astrolabe. The zenith distance is set at any convenient angle 40°, 50°, 60°, 70°.

Observations can be made with fixed horizontal reticle lines, or with an impersonal micrometer. In the first case a reticle has several closely spaced horizontal lines ruled on glass, as shown in the figure 2, and the passage of each star over them is recorded.

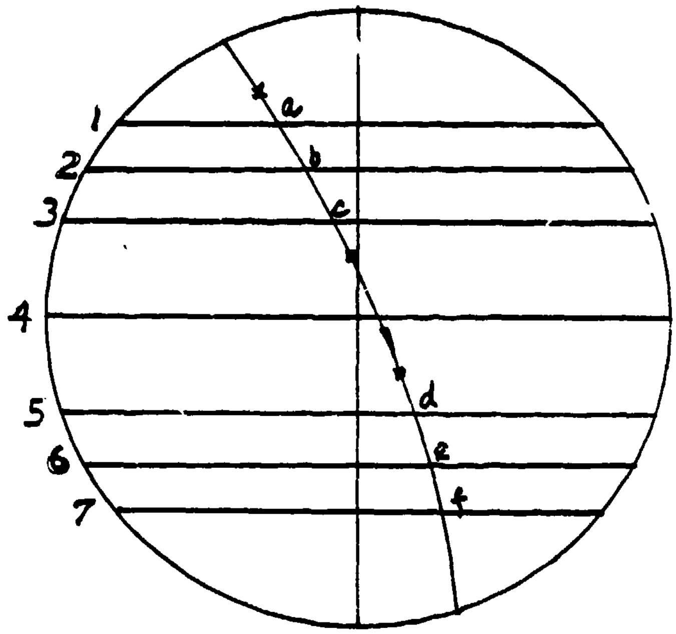

Figure 2 showing the reticle horizontal lines, and the trajectory of a star, whose transit times are recorded at a, b, ...f.

If the theodolite has a moving wire impersonal micrometer, it is essential to use the same contacts on all stars observed at equal zenith distance, since the movement of the micrometer will be in opposite directions for east and west stars, so corrections must be made for width of contacts and lost motion.

In a reticle, the lines are almost symmetrical with respect to the central line. Using an impersonal micrometer's symetrical pair of contacts can be considered too.

Consider two symmetrical lines or micrometer contacts. These two lines have a mean zenith distance very close to

that of the central wire. We shall consider any pair of lines independent of one to another, so we can consider several independent results of longitude determination.

Let Z_0 be the zenith distance that corresponds to any pair. To this zenith distance will correspond a time θ_0 which must be determined from the times of a star's passages over these two lines, since the star trajectory is not a straight line. The transit times are a function of the zenith distances, so according to Taylor's theorem we have

$$\theta_i = f(Z_i) = f(Z_0 + \Delta Z_i)$$

$$\theta_i = \theta_0 + \left(\frac{d\theta}{dZ}\right)_0 \Delta Z_i + \frac{1}{2}\left(\frac{d^2\theta}{dZ^2}\right)_0 \Delta Z_i^2 + \frac{1}{6}\left(\frac{d^3\theta}{dZ^3}\right)_0 \Delta Z_i^3. \quad (47)$$

where

$$\Delta Z_i = Z_i - Z_0$$

Let

$$i = 1, \text{ first line}$$
$$i = 2, \text{ second line.}$$

then

$$\Delta Z_1 = Z_1 - Z_0 = -\frac{1}{2}(Z_2 - Z_1)$$
$$\Delta Z_2 = Z_2 - Z_0 = \frac{1}{2}(Z_2 - Z_1). \quad (48)$$

Inserting these values into equation (47), we have for the mean value:

$$\frac{1}{2}(\theta_1 + \theta_2) = \theta_0 + \frac{1}{8}\left(\frac{d^2\theta}{dZ^2}\right)_0 (Z_2 - Z_1)^2. \quad (49)$$

Then the time that corresponds to the mean zenith distance Z_0, is

$$\theta_0 = \frac{1}{2}(\theta_1 + \theta_2) - \frac{1}{8}\left(\frac{d^2\theta}{dZ^2}\right)_0 (Z_2 - Z_1)^2. \quad (50)$$

The value of $(Z_2 - Z_1)$ is unknown. To evaluate it we proceed as follows:

Consider the zenith distance as a function of time. A zenith distance Z_x will now correspond to the mean time of the star passage over the two lines.

Applying Taylor's theorem we have for upper and lower horizontal lines:

$$Z_1 - Z_x = -\frac{dZ}{dt}\Delta\theta + \frac{1}{2}\frac{d^2Z}{dt^2}\Delta\theta^2 - \frac{1}{6}\frac{d^3Z}{dt^3}\Delta\theta^3,$$

$$Z_2 - Z_x = \frac{dZ}{dt}\Delta\theta + \frac{1}{2}\frac{d^2Z}{dt^2}\Delta\theta^2 + \frac{1}{6}\frac{d^3Z}{dt^3}\Delta\theta^3 \quad (51)$$

where

$$\Delta\theta = \frac{1}{2}(\theta_2 - \theta_1).$$

From (51) there follows the difference:

$$Z_2 - Z_1 = \frac{dZ}{dt}(\theta_2 - \theta_1) + \frac{1}{24}\frac{d^3Z}{dt^3}(\theta_2 - \theta_1)^3. \quad (52)$$

The second term on the right side of equation (52) is very small since $(\theta_2 - \theta_1)$ does not reach 4 minutes of time, therefore we may consider without noticeable error that

$$Z_2 - Z_1 = \left(\frac{dZ}{dt}\right)^2 (\theta_2 - \theta_1)^2. \qquad (53)$$

Whence with the help of this equation where we can compute according to equation (50), the value of θ_0 as follows:

$$\theta_0 = \frac{1}{2}(\theta_2 + \theta_1) - \frac{1}{8}\left(\frac{d^2 t}{dZ^2}\right)_0 \left(\frac{dZ}{dt}\right)_0^2 (\theta_2 - \theta_1)^2. \qquad (54)$$

From consideration of the definition of sidereal time, hour angle, and right ascension, the first equals the sum of the other two, that is:

$$\theta = \alpha + t$$

so we have

$$\frac{d\theta}{dZ} = \frac{dt}{dZ}. \qquad (55)$$

A formula for computing $\frac{d^2\theta}{dZ^2}$ can be obtained in the following way:

By differentiating the general relation

$$\cos Z = \sin\phi \sin\delta + \cos\phi \cos\delta \cos t \qquad (56)$$

with respect to Z, considering δ and ϕ constants, we have:

$$\sin Z = \cos\phi \cos\delta \frac{dt}{dZ} \sin t. \qquad (57)$$

A second differentiation gives:

$$\cos Z = \cos\phi \cos\delta \cos t \left(\frac{dt}{dZ}\right)^2 + \cos\phi \cos\delta \sin t \frac{d^2 t}{dZ^2}. \qquad (58)$$

By replacing the coefficients of the differentials in equation (58) from the values obtained from the equations (56) and (57), and after multiplying by $\left(\frac{dZ}{dt}\right)^2$, the resulting equation for $\frac{d^2 Z}{dt^2}$, we obtain

$$\frac{d^2 Z}{dt^2}\left(\frac{dZ}{dt}\right)^2 = \frac{\left[\cos Z - (\cos Z - \sin\phi \sin\delta)\left(\frac{dt}{dZ}\right)^2\right]}{\sin Z}\frac{dZ}{dt}, \qquad (59)$$

in which the only unknown to be evaluated is $\frac{dZ}{dt}$.

From the relation,

$$\cos\delta \sin t = \cos\phi \sin A, \qquad (60)$$

equation (57) gives

$$\frac{dZ}{dt} = \cos\phi \sin A. \qquad (61)$$

To evaluate sin A we use the equation

$$\cos A = P - Q \sin\delta, \qquad (62)$$

where P and Q are constants,

$$P = \tan\phi \cot Z \qquad (63)$$

and

$$Q = \sec\phi \operatorname{cosec} Z.$$

Squaring and subtracting from unity we have:

$$\sin A = \pm\sqrt{1-(P-Q\sin\delta)^2}, \tag{64}$$

whence it follows:

$$\frac{dz}{dt} = \pm\cos\phi\sqrt{1-(P-Q\sin\delta)^2}. \tag{65}$$

Inserting this value in equation (61) and the resulting value in equation (56), we finally obtain:

$$\theta_0 = \frac{\theta_1+\theta_2}{2} - \frac{1}{8}\left[M\cos\phi\cot z - \frac{(\cos z - \sin\phi\sin\delta)}{M\cos\phi\sin z}\right](\theta_2-\theta_1)^2 \tag{66}$$

where

$$M = \pm\sqrt{1-(P-Q\sin\delta)^2} . \tag{67}$$

The positive sign is for the star west of the meridian and negative sign for the star to the east.

Example of Computation

As an example we give the results from data taken from Niethammer reference 4, page 60-61.

Our method shows a discrepancy of $0^{s}.106 = 1''59$ with respect to Niethammer's solution.

TABLE 1. Clock Correction (Longitude) For An Equal Altitude Star Pair Transit Times

		West Star	East Star
1	θ	$17^h 55^m 14^s.020$	$17^h 50^m 02^s 600$
2	dt_i^*	+0.092	-0.090
3	α	14 29 25.280	21 10 35.500
4	$d\alpha^{**}$	+0.010	+0.010
5	δ	30 37' 11".20	30 00' 01".24
6	$d\delta$	-0.09	+0.08
7	$\delta_w, \delta_e = 5+6$	30 37' 11".11	30 00" 01".32
8	$\beta_w, \beta_e = (1+2)-(3+4)$	$3^h 25^m 48^s.822$	$-3^h 20^m 33^s.000$

9 $\beta_0 = \frac{1}{2}(\beta_w - \beta_e)$ $3^h 23^m 10^s.911$ = 50° 47' 43".66

$$\tan F = \frac{\cos\delta_w - \cos\delta_e}{\cos\delta_w + \cos\delta_e}\cot\beta_0$$

10 tan F = -0.00 257751

11 F = -0° 08' 51"65 = ...-35^{s}443

12 $\frac{1}{2}(\delta_w - \delta_e)$ 0° 18' 34".90

13 ϕ 47° 32' 27"

$\sin(\gamma - F) = \tan\frac{1}{2}(\delta_w - \delta_e)\tan\phi \operatorname{cosec}\beta_0$

14 sin(γ - F) = 0.00762324

15 $\Delta T = \gamma - F$ = 0° 26' 12"42 = ...1^m 44.828

-2^m 37.911

16 ΔT = 11+15+16 . . . . -1^m 28^s.526

Niethammer ref. 4 gives . . . . -1^m 28^s.420

Discrepancy N-B . . . . 0^s.106=1"59

* Correction for inclination error

** Correction for aberration

REFERENCES

1. Baldini, Angel A. 1963. "Method of Independent Equations for Determining Latitude, Longitude and Zenith Distance," presented at the XIIIth General Assembly of the International Union of Geodesy and Geophysics, Berkeley, California.

   "Determination of Level Sensitivity Without Moving the Level from the Instrument," Survey Review XXIII, 175, January 1975.

2. Barnes, G. L. and I. I. Mueller. 1966. "The Dependence of the Level Sensitivity on the Position and Length of the Bubble on the Wild T-4 Theodolite." Bulletin Geodesique.

3. Kivioja, L. A. 1979. "New Mercury Instrument," Surveying and Mapping, Vol. XXXIX.

4. Niethammer, TH. 1947. Die genauen Methoden der Astronomisch-geographischen Ortsbestimmung. Verlag. Birkhauser. Basel.